Further Performance Tests on the CSIRO Liquid Water Probe

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ABSTRACT

A further 400 h of flying experience with the CSIRO hot-wire probe has shown that it can accurately measure liquid water content in clouds. Computations and experiments suggest that when an epoxy coating is used for protection, it should be less than 50 μm thick, and that the wire should be operated around 160°C when such coatings are used. Comparisons of performance with the Axially Scattering Spectrometer Probe and in a wet wind tunnel indicate that splashing of drops up to 40 μm diameter is not a problem at speeds up to 80 m s^{-1}.

1. Introduction

The CSIRO hot-wire probe is a device for measuring cloud liquid water content (LWC) from aircraft. The principle of operation is the measurement of the power required to maintain the temperature of a hot wire on which cloud droplets are impacting. It is probable that the probe will be widely used for such measurements since it is robust, reliable and accurate, and it is important therefore that all aspects of its performance are understood. The basic design, the electronic control, and an additional use of the probe have been described in papers by King *et al.* (1978), Bradley and King (1979) and King and Handsworth (1979), henceforth referred to as I, II and III. Since Paper I was published, a further 400 flying and 600 laboratory hours of experience have been accumulated. This paper describes details of the probe performance which have emerged as a result of that experience and also describes two further calibration checks on the probe at aircraft speeds.

2. Wire temperature and its effect on performance

a. *Influence of the offset voltage*

In I it was stated that the hot-wire temperature is determined by the bridge resistors and lead wire resistances. This result was obtained by assuming that the offset voltage introduced into the bridge for circuit stability purposes (see I and II) has a negligible effect on the operating point of the bridge. We now show that under normal operating conditions the maximum error from this cause is quite acceptable.

In II it was shown that when operating under steady-state conditions the out-of-balance bridge voltage V_{in} is given by

$$V_{in} = \frac{V_s(nR_1 - R)}{(n + 1)(R_1 + R)} + \frac{nR_2V_0}{(n + 1)R_0}, \quad (1)$$

where n is the bridge ratio, V_s and V_0 are the bridge and offset voltages, respectively, R_1 and R_2 are bridge resistors, and R_0 and R are offset feed and hot-wire resistances, respectively. Now $V_s = GV_{in}$, where G is the feedback amplifier gain, leading to

$$(nR_1 - R) = (n + 1)(R_1 + R)G^{-1} - n(R_1 + R)R_2V_0(R_0V_s)^{-1}, \quad (2)$$

where the left-hand side of (2) represents the difference between the "set" hot-wire resistance [as defined by (2) in Paper I] and the actual operating resistance. For typical values of $R_1 = 1\Omega$, $n = 4.6$, $R = 4.6\Omega$ and $G = 10^3$, the first term on the right-hand side is $\sim 0.03\Omega$ and the second term $0.12/V_s\Omega$, provided the offset is set for critical damping as described in II. Therefore the difference between set and operating resistances is less than $\pm 0.02\Omega$ for V_s in the range 2.5–10V (i.e., from still air to typical flying conditions), which corresponds to a temperature difference of ±1°C about the set point temperature. In other words, the offset voltage introduces a maximum error of ~1°C. Although this is quite acceptable, this conclusion is based on the assumption that the offset voltage is set up as recommended in II, and that the amplifier constants τ_1 and G^{-1} are reasonably small.

b. *Dry term*

There are two contributions to the total heat dissipated by the hot wire. The first arises from forced convection of dry air moving past the heated wire, and this amount needs to be known before the second contribution, which is associated with

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evaporation of the cloud droplets, can be calculated. Wind-tunnel tests described in I established that a relationship of the form

$$\mathrm{Nu} = A(\mathrm{Re})^x, \tag{3}$$

where Nu and Re, the Nusselt and Reynolds numbers, respectively, describe the dry heat losses well. In (3) A is a constant, and in the following equations the symbol A with subscripts will be used to denote unknown constants (to be determined from the real data) which are proportional to the A of (3). Although the value of x determined by these tests was in good agreement with theoretical expectations, A was ~20% higher than expected, probably because of turbulence, and users of the wire were advised to use actual flight data to establish values for A and x which gave best predictions to the dry heat losses. In terms of quantities which are actually measured, Eq. (3) can be written as

$$P_d = A_1 k_a (T_w - T_a)(vD\rho/\eta)^x, \tag{4}$$

where P_d is the power delivered to the wire under dry conditions, T_w and T_a are the temperature of wire and air, respectively, k_a, ρ and η are the thermal conductivity, density and viscosity of air at the film temperature $(T_w + T_a)/2$, v is the true air velocity, and D the diameter of the wire. In plotting aircraft data in a form that allows A_1 and x to be estimated, it is important to allow for variations in k, ρ and η that arise from altitude changes. Fortunately, both k and η do not vary with pressure, and their temperature dependence can be described by

$$k(T) = k(273)\cdot(398/125 + T)\cdot(T/273)^{3/2}, \tag{5}$$

$$\eta(T) = \eta(273)\cdot(393/120 + T)\cdot(T/273)^{3/2}, \tag{6}$$

where T is temperature (K) (International Critical Tables, Vol. 5, pp. 1 and 213). Substituting these expressions into (4) and allowing for the fact that ρ varies as p/T leads to

$$P_d \approx A_2 (T_w - T_a) v^x p^x T^{(3-5x)/2} \frac{(120 + T)^x}{(125 + T)}. \tag{7}$$

Now we have found for our hot wires that $0.50 < x < 0.65$. Further, for $T_w = 150°\mathrm{C}$ (see later) and T_a in the range −20°C to +20°C, then $(T_w + T_a)/2$ varies only in the range 343–363 K, so that the maximum variation in the last three terms of (7) amounts to no more than 2.1%, and is substantially less than this as $x \rightarrow 0.6$. Neglecting this variation then leaves us with $P_d \approx A_2(T_w - T_a)(vp)^x$, and if T_w is known exactly A and x can be obtained by linear regression of $\log[P_d/(T_w - T_a)]$ against $\log(vp)$. However, for many reasons, one of which was mentioned in Section 2a, it is unlikely that T_w will be known to an accuracy of better than a few degrees, and in these cases the prediction of P_d can best be achieved by a nonlinear regression in which T_w, A_2 and x are all estimated. If this technique produces a best estimate of T_w far removed from the set temperature, it should provide cause for concern about the way in which the bridge was set up, but it need not detract in the least from the usefulness of the expression used to predict P_d, provided that sufficient of the variance is accounted for.

An example of aircraft data analyzed in this manner is shown in Fig. 1. The ordinate is proportional to $P_d/(T_w - T_a)$ and the abscissa to $(vp)^{0.515}$. The regression curve accounts for 98% of the variance, and the maximum error in predicting P_d is less than 0.02 g m^{-3} in terms of equivalent LWC.

c. Effect of an epoxy coating

In I it was suggested that wires to be used in clouds containing graupel should be coated with a thin layer of epoxy to make them more robust. Experience has shown that this advice is useful—during 1979 four Australian aircraft flew hot-wire units for a total of 430 h with damage to only two elements. A caution was added in I to the effect that the coating should be thin enough to allow its surface temperature to be close to the wire temperature, but no guidelines were given as to what a reasonable thickness would be. In this section we examine the effects of the epoxy coating and give estimates of what the maximum thickness should be.

1) INFLUENCE OF AN INSULATING LAYER ON THE DRY TERM

Consider the coated hot wire as a cylinder of radius a at temperature T_w surrounded by a thickness t of insulating material of thermal conductivity k_i, and losing heat to the environment at temperature T_a according to (3). Then from Carslaw and Jaeger (1959, p. 189), we find that the temperature T_s of the surface of the insulator is given by

$$T_s = \frac{[T_w + \lambda T_a (a + t) \log(1 + t/a)]}{[1 + \lambda(a + t)\log(1 + t/a)]}, \tag{8}$$

where

$$\lambda = \mathrm{Nu} k_a / 2(a + t) k_i. \tag{9}$$

Now since $t \ll a$, then (8) reduces to

$$T_s = (T_w + \lambda t T_a)/(1 + \lambda t), \tag{10}$$

which can be rearranged further to give

$$(T_w - T_s)/(T_w - T_a) = \lambda t/(1 + \lambda t). \tag{11}$$

Thus if we wish the surface temperature cooling to be less than 10% of the difference between the hot-wire and air temperatures, then $\lambda t < 0.11$. At speeds of 70 m s^{-1}, Nu ≈ 60 for the hot wire, and using (9) with $k_i = 0.22$ J m^{-1} s^{-1} $°\mathrm{C}^{-1}$, t should therefore be less than about 30 μm. In practice,

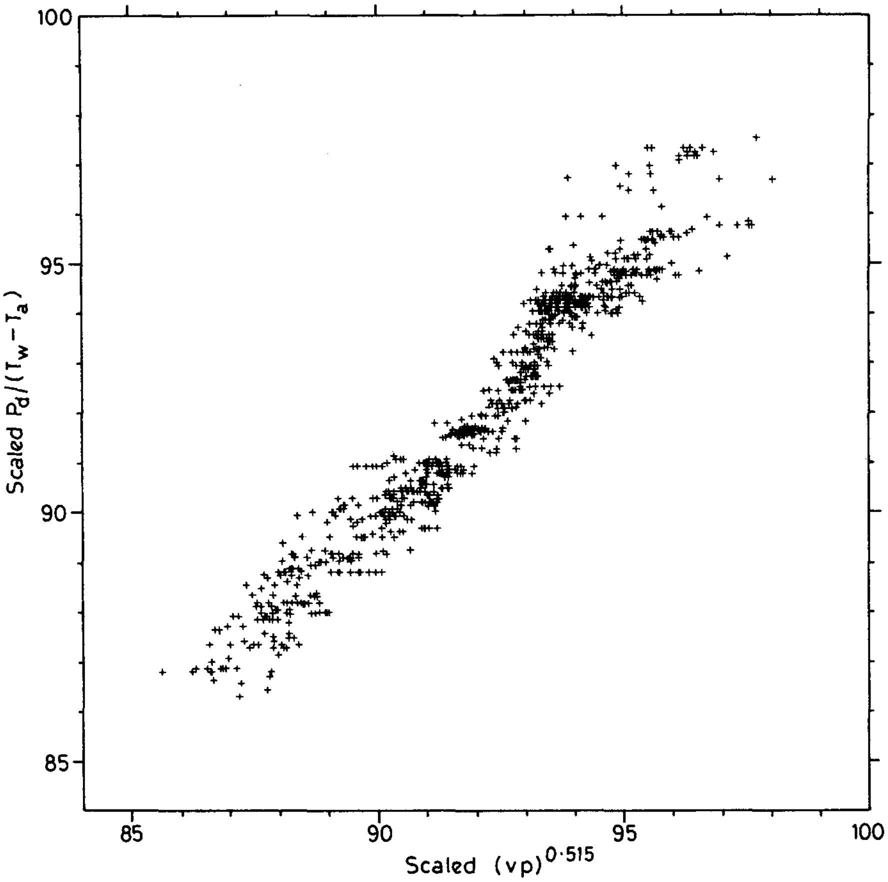

FIG. 1. Aircraft data for hot-wire behavior in clear air. The ordinate is proportional to $P_d/(T_w - T_a)$ for T_w = 122°C, and the abscissa is proportional to $(vp)^x$, for $x = 0.515$. Both T_w and x were determined by nonlinear regression. The maximum deviation from the least-fit line is equivalent to an LWC uncertainty of ∓ 0.02 g m^{-3}.

a coating of this thickness is relatively easy to apply, yet it still adds considerable strength. It is important, however, that the thickness be measured, for although coatings of 100 μm still appear very thin, they can cause a surface cooling of up to 30°C.

Although the epoxy coating reduces the surface temperature, it produces little change to the form of the relationship described in (3) and (4). With the surface temperature of the coating at T_s, the power lost will vary as

$$P_d \approx A_3(T_s - T_a)\,\mathrm{Re}^x \qquad (12)$$

and substituting from (10), we have

$$P_d \approx A_3(T_s - T_a)\,\mathrm{Re}^x/(1 + \lambda t). \qquad (13)$$

Although λ varies as Re^x through (9), the dependence on Reynolds number is essentially the same as in (13), provided λt is small. Thus the power lost from a coated wire is simply reduced by the factor $(1 + \lambda t)$.

One feature which does arise from (13) is that the effects of a thick coating will not show up as a lower T_w in any nonlinear regression treatment such as mentioned in Section 2b but rather as a reduction in the slope of the best-fit line at higher Reynolds numbers.

2) EFFECT OF AN INSULATING LAYER ON WET PERFORMANCE

Given that a thickness of only 30 μm can cause the surface temperature of the probe to be several degrees colder than the wire under dry conditions, the more localized heat load associated with the evaporation of cloud droplets would be expected to cause a more pronounced effect. Unfortunately, however, we have been unable to derive a suitable quantitative analysis which describes the evaporation of a droplet on an insulating film and which agrees with the data we will present later, so our arguments at this stage are qualitative only.

For a wire of 1.7 mm diameter moving through a cloud at 60 m s^{-1}, the wet and dry heat losses are about equal for an LWC of ~1 g m^{-3}. Therefore, the average heat flux over the whole wire surface is doubled by the impaction of drops that make up an LWC of 1 g m^{-3}. We consequently would expect that if this liquid water were spread uniformly over the wire, the cooling of the surface could be estimated fairly accurately by replacing λ with 2λ in (10), since λ is proportional to the heat flux. Now the wire is unlikely to be covered uniformly by these drops, and probably has less than 10% cover at 1 g m^{-3} (see the Appendix of I), so that the heat flux under the drops must be of the order of 10 times the heat flux on the dry parts of the wire. Consequently, a more realistic estimate of T_s under a drop can be obtained by replacing λ with 10λ in (10). (This argument overestimates the magnitude of the cooling because it ignores any heat transfer along the surface of the probe, but it does give a worst-case estimate.) Using (10), and taking T_w = 100°C, T_a = 0°C, we find that T_s could be as low as 45°C for a 30 μm coating. For the same conditions but with T_w = 160°C, we have T_s = 75°C, which is much closer to the desired temperature of ~90°C.

As an experimental test of these ideas, large drops were placed on the top surface of a horizontal hot wire with a 50 μm coating and their evaporation times measured. The experimental procedure was to place a drop on the hot wire with a syringe, measure its diameter with an eye scale, and time the evaporation. Drop diameters in the range d = 0.8–2.5 mm were used. Experimental variance arose because the drop shape was not constant over the size range used (the smaller drops were almost hemispherical, the larger ones flattened spheroids) and for a given mass the shape depended somewhat on the way the drop was placed on the wire. Further, as the drops evaporated the area in contact with the wire was substantially constant for ~90% of the evaporation time as the drop thinned out, and its diameter only changed during the last stages. It is this feature which makes it so difficult to provide an analytic description of the process.

Four separate series of measurements were taken with the wire at temperatures from 111 to 160°C. Only for T_w = 160°C was any nucleate boiling within the drop observed. At the cooler temperatures, the drops steadily evaporated. Of course, this means that effective drop temperatures were considerably less than 100°C. The results are shown in Fig. 2, along with the curve obtained from the hemispherical cap model described in I for a temperature of 100°C. (This model assumes a constant temperature throughout the drop, and does not allow for ventilation effects.) Two features which emerge from Fig. 2 are as follows:

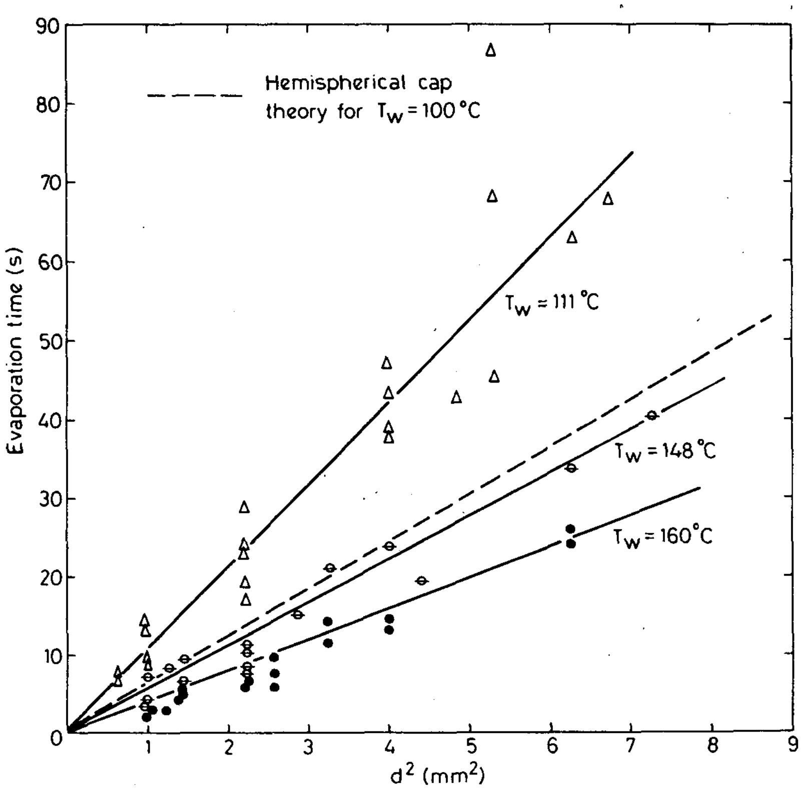

FIG. 2. Evaporation times for drops of diameter d placed on a hot wire with a 50 μm epoxy coating.

1) The evaporation times increase as d^2 for all wire temperatures. This is rather surprising, since a d^2 dependence is characteristic of the hemispherical cap model and suggests a constant drop temperature during the evaporation process.

2) For $T_w = 160$ and 148°C the drops evaporate faster than would be expected from the hemispherical cap model for drops boiling during the entire evaporation process. This suggests that the combined effects of ventilation and a larger surface area are important in decreasing the time required to evaporate the drops. (In fact, the only way in which this data can be reconciled to the hemispherical cap model is to take a constant ventilation factor of ~1.5 for all drops and to take the droplet temperature as being 0.6 of the wire temperature.) It should be noted that these measurements were taken with ventilation of the drops arising only from free convection. The forced ventilation due to aircraft motion will act to reduce the evaporation times so that the above measurements are worst-cast estimates of the problem.

We do not wish to place too much emphasis on these rather crude measurements, especially since they apply only to one particular coating thickness, but the following points should be noted: 1) there are reasonable arguments and experimental data indicating that droplet temperatures will be considerably lower than the wire temperature; 2) the main reason for not operating the wire at temperatures well in excess of 100°C is that a vapor film could form under the drop and slow down the evaporation. We found no evidence of this occurring on a wire with a 50 μm coating operated at 160°C, and, consequently, we would recommend 160°C as a suitable operating temperature. (At higher temperatures the epoxy hardens and degrades considerably.) In Section 3c we present data showing that provided the temperature is around 160°C the frequency response of the probe is not adversely affected when operated with such an epoxy coating.

There is one further consideration which relates to the main operating equation of the wire. The power due to the wet term is

$$P_w \approx [L + C(T_e - T_a)]wlvD,$$

where L and C are the latent and specific heats of water, T_e and T_a are the evaporation and ambient temperatures, v the true air velocity, w the LWC, and l and D the dimensions of the wire. When the wire is bare, the relative thermal conductivities of air and copper are such that T_e can be taken as T_w, but this is no longer true when the wire is coated and $T_e \ll T_w$. Fortunately, and not unexpectedly, the sum $L(T_e) + C(T_e - T_a)$ is fairly constant for $T_a \approx 0$°C and T_e in the likely range of interest. In particular, if one takes T_e as 80°C the maximum variation in $L + C(T_e - T_a)$ is less than $\mp 1.5\%$ for $60°\text{C} < T_e < 100°\text{C}$.

3. Operation and calibration at aircraft speeds

The original calibrations of the wire as described in I were all carried out at speeds < 20 m s^{-1}. While the aircraft results described in III indirectly confirm the possibility of using the device at higher velocities, a separate direct calibration was deemed necessary, particularly since it seemed possible that at higher speeds the larger cloud droplets could splash off the wire before evaporating. The following sections deal with measurements and comparisons at speeds up to 90 m s^{-1}.

a. Comparisons with ASSP

Fig. 3 shows a comparison between hot-wire-derived LWC's and those obtained by integrating over the droplet spectra obtained from the ASSP optical scattering probe.[1] Two-second averaging times were used for both instruments. Two of these comparisons were for maritime clouds with median drop diameters of ~12 and 24 μm while the third was obtained in more continental clouds. The ASSP data have been corrected for coincidence losses arising from the dead time of the instrument. Even though this dead time has been determined from both laboratory and in-flight data, the errors and uncertainties involved in making such adjustments are quite large, to the extent that the error involved in the liquid water measurement from the continental spectrum was ~100%, as compared with ~30% for the maritime spectra. (These rather large errors are also partly due to uncertainties in the size calibration and sampling volume of the ASSP.) Thus while the agreement in Fig. 3 is quite reasonable, and in no way suggests that large droplets are not "seen" by the hot wire, the large errors in the ASSP-derived liquid water detracts from the value of any such comparisons, and highlights the difficulties in measuring moments of a distribution from the distribution itself.

b. Wet wind tunnel tests

The basis of this test is the injection of a mass of water in droplet form down a wind tunnel at a known rate. Provided the air and water droplets are well mixed, the response of the hot wire is calculable in terms of the mass flow rate and the dimensions of the tunnel, irrespective of the velocity of air (provided, of course, that the velocity is high enough for the droplets to be collected).

To achieve a reasonably uniform LWC during the experiments the normal bell-shaped entrance to a small (13 cm diameter) wind tunnel was removed,

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[1] The Axially Scattering Spectrometer Probe is manufactured by Particle Measuring Systems, Boulder, Co.

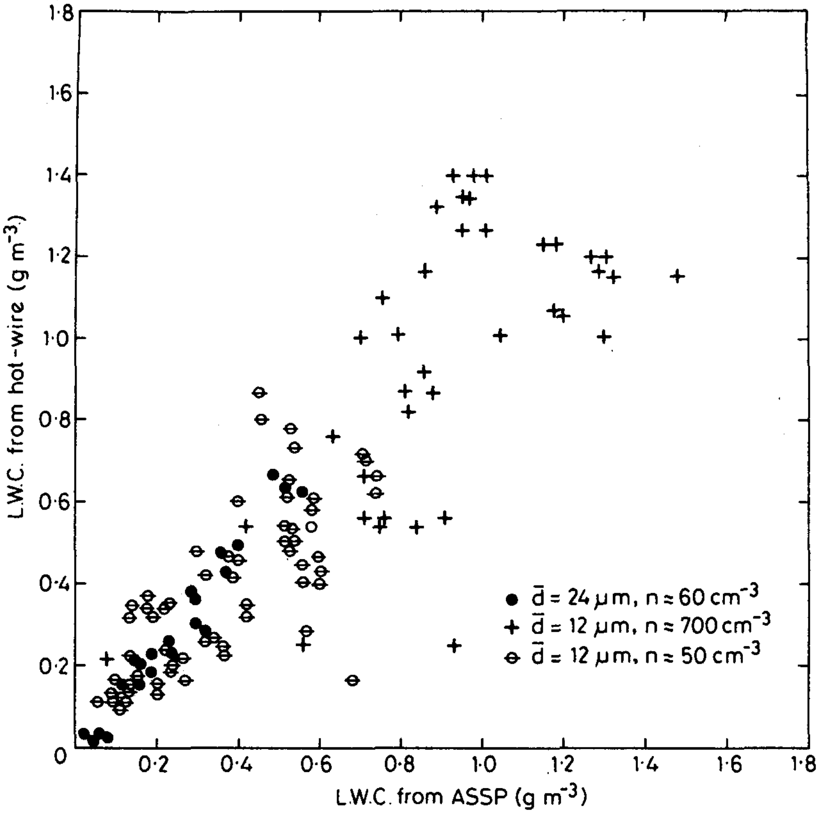

FIG. 3. Comparison of LWC's derived from the hot wire and the ASSP. The line of least fit has a slope of 1.1 ∓ 0.1.

and a cone of half-angle 30° and inlet diameter 1 m was substituted. Water was injected from five similar nozzles fed from a common water source whose mass was monitored on a beam balance. The purpose of the wide entrance was to allow sufficient mixing across the width of the flow while the air was still moving relatively slowly. The flow rate from the nozzle was kept constant, and was equivalent to 1 g m^{-3} at 80 m s^{-1}, with correspondingly more at the lower speeds.

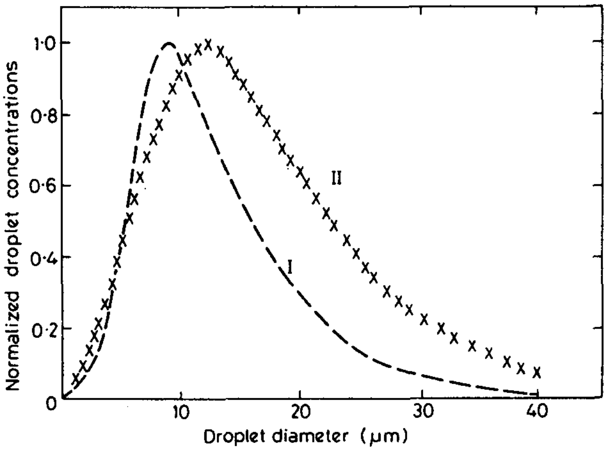

FIG. 4. Two different droplet spectra generated from sprays and used in the wind-tunnel tests. The mean volume weighted diameters for I and II were 15 and 23 μm, respectively.

By varying the nozzle diameters the mean droplet size could be altered substantially. The droplet size spectra as measured by the ASSP for the two nozzles used are shown in Fig. 4. Both produced significant liquid water in droplet sizes > 40 μm.

Droplet evaporation was initially a serious problem in these experiments. If the wind tunnel air is subsaturated by more than a few percent, a significant fraction of the mass of cloud droplets can be lost by evaporation even in the few tens of milliseconds that it takes to travel down the tunnel. Such losses cause large errors, and to decrease their magnitude the experiment was performed, with the conical entrance of the wind tunnel placed in the outside air, only on those days on which considerable rain was falling.

Many attempts were made to assess the transverse spatial uniformity of the cloud moving down the tunnel using soot slides, etc., but none of these was successful. The only evidence we have that good mixing was achieved comes from the insensitivity of the results to changes in the position and orientation of the hot wire, and to changes in the axial and transverse positions of the nozzles. Visually the cloud seemed to consist of discrete lumps that travelled down the tunnel along different

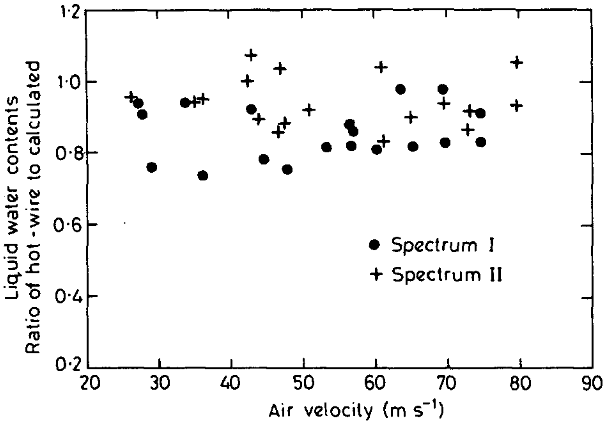

FIG. 5. The ratio of LWC from the hot wire to the LWC injected into the tunnel as a function of airspeed. There is no significant decrease with airspeed for either spectrum, and the slightly lower values for the small-drop spectrum suggests that evaporation may have been a problem.

axes, depending on the strength of cross breezes at the entrance, but there did not appear to be any preferred axis.

The experimental procedure was to run the tunnel at the desired speed, obtain a measure of the dry term, and then turn on the nozzles for ~2 min. The total mass of water emitted from the nozzles was noted, and any water droplets which sedimented out into the cone were mopped up and weighed. (These losses were typically ~3% of the total mass going down the tunnel.) The liquid water as seen by the hot wire was averaged for the 2 min period (signal variations amounted to ~30% due to the discreteness of the cloud). This procedure was then repeated at different speeds and with different nozzles.

The results of these tests are shown in Fig. 5. Despite the scatter it is clear that for both droplet spectra there is no obvious decrease in response at the higher speeds. This evidence suggests fairly strongly that, at least for cloud droplets up to 40 μm diameter, splashing is not a serious problem at speeds up to 80 m s^{-1}. What is apparent from Fig. 5 is that the mean value of LWC as seen by the hot wire is about 0.9 of that calculated as going down the tunnel and that the response to the smaller drops is slightly less than that for the larger drops (just the opposite effect to what would be expected if splashing were involved). Possible reasons for these differences include the following:

1) Although the tests were conducted on rainy days, dew point depressions of a few tenths of a degree were measured in the inflow air. This is sufficient subsaturation for a 10 μm droplet to lose 7% of its mass going down the tunnel.

2) Sedimentation in the cone was allowed for but sedimentation in the tunnel itself was not. The magnitude of this loss could not be measured accurately but was estimated to be less than 2%.

3) Droplets from the nozzles were being blown across the entrance and were not entering the tunnel. The magnitude of this effect is unknown.

It thus seems possible that water losses could account for some, if not most, of the 10% difference found, and that the overall response is neither size- nor velocity-dependent for droplets up to 40 μm and speeds up to 80 m s^{-1}. In the light of these experiments, perhaps the only way in which to perform a better calibration would be to collect rime from a natural supercooled cloud, or refrigerated wind tunnel.

c. *High-speed response*

In Fig. 6, we present a power spectrum of LWC obtained from a single penetration of a medium size cumulus 1200 m above cloud base. The important feature of this curve, from an instrumental performance point of view, is the smooth decrease from 45 to 3 m wavelength with a slope of −1.8. [This slope is greater than one would expect for homogeneous turbulence, but less than the slopes of about −2 found by Warner (1970) for updraft power spectra.] The main point of interest here is that if the instrument had a limiting frequency response somewhere in the range from 2 to 30 Hz, then one would expect this to show up as a further slope of −2 imposed on these curves beyond the cutoff frequency. Since there is no evidence of this, there is good reason to believe that the probe performs satisfactorily at least up to 30 Hz even with the epoxy coating.

Because the probe has such a rapid response its output needs to be suitably filtered before recording where the sampling rate is ≤50 Hz. It is important that this filtering be performed on the signal corresponding to the product function proportional to the power delivered to the hot wire, and not to the bridge voltage or something similar, since the average of a product is necessarily greater than the product of the corresponding averages. Experience has shown that a convenient way around this problem is to use a fast analogue multiplier to perform the power computation, followed by a filter with a time constant appropriate to the sampling rate.

4. Conclusions

Further experience with the CSIRO hot wire extending over two years and 400 flying hours has shown that the instrument is robust, reliable and accurate. Any epoxy coating used should be ≤50 μm thick, and the wire should be operated around 160°C when such coatings are used. This temperature is almost twice that recommended in Paper I for coating-free wires. Comparisons of performance with the ASSP and in a wet wind-tunnel indicate

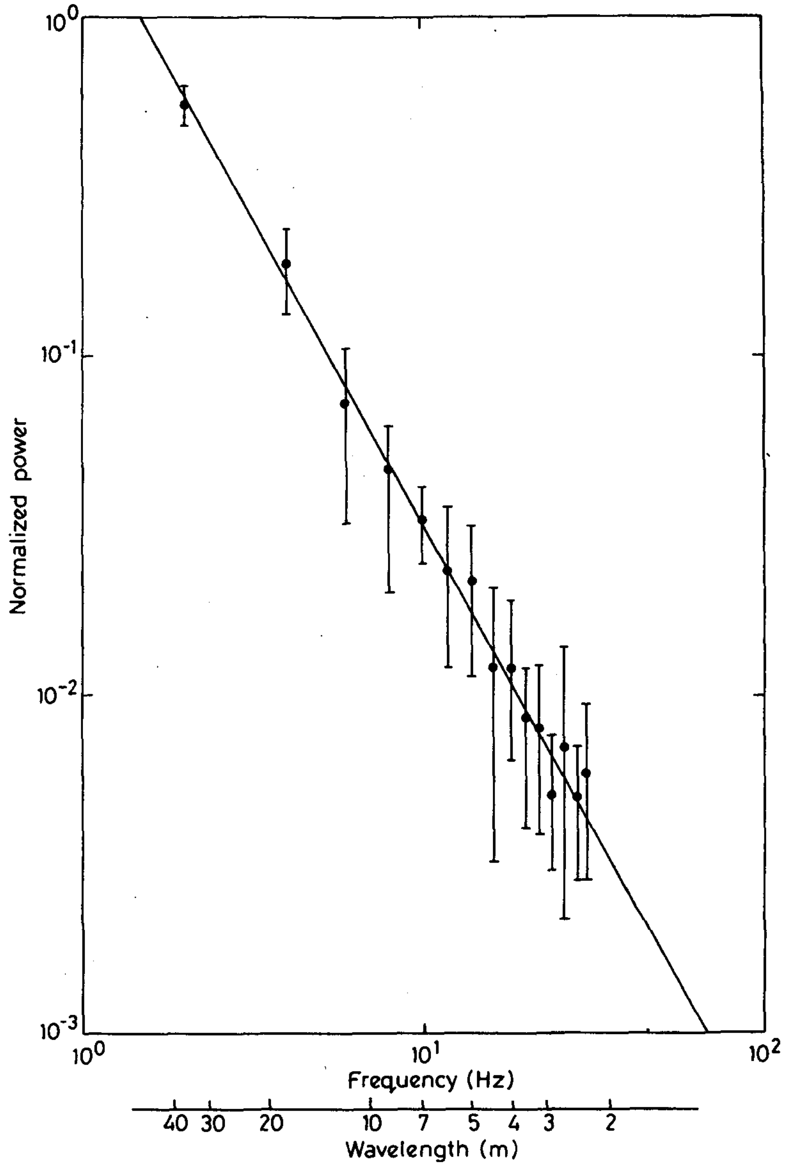

FIG. 6. Normalized power spectrum for LWC obtained from a single penetration of a medium cumulus. The smooth roll-off with a slope of −1.8 up to 30 Hz suggests that the probe response is not affected by the epoxy coating.

that splashing of drops up to 40 μm is not a serious problem at 80 m s^{-1}. It would be interesting to know how the probe performs on jet research aircraft but extrapolation would appear to be risky in view of the fourfold increase in kinetic energy of the drops.

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