A Hot-Wire Liquid Water Device Having Fully Calculable Response Characteristics

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ABSTRACT

A liquid water sensor consisting of a thin copper wire wound on a hollow 1.5 mm diameter cylinder is described. Slave coils on either side of the master sensing coil reduce axial heat losses to an acceptable level, and allow for a simple relationship between power supplied to the wire and liquid water content. Wet wind-tunnel tests show that the system response to liquid water is easily calculable from a knowledge of the geometrical dimensions of the cylinder and the operating temperature of the hot wire. When operated at 100°C, the device has a sensitivity of 0.02 g m^{-3}, a response time of better than 0.05 s and an accuracy of 5% at 1 g m^{-3}.

1. Introduction

The liquid water content of clouds is still one of the more fundamental variables that cloud physicists attempt to measure. Its distribution and evolution are relevant in cloud seeding, cloud modeling and cloud microphysics. Of the instruments currently available to measure liquid water, the Johnson-Williams[1] hot wire is the most widely used. In this instrument a constant current passing through a wire exposed to the airstream heats the wire, and its temperature is monitored to give a measure of the liquid water. A similar wire which is kept dry is used to compensate for the heat supplied to the air moving past, although the validity of this procedure has been questioned by Merceret and Schricker (1975). In a "nimbiometer" designed by them a hot wire is maintained at constant temperature and the power monitored. Although this more readily permits analytic treatment, this instrument, like the Johnson-Williams, still requires either a wet wind tunnel or another liquid water instrument for calibration. Devices which yield the full droplet size distribution, such as the Knollenberg optical scattering probe[2] or the older soot slides are also used to determine liquid water by integration of the droplet spectrum. The former is expensive, technically sophisticated, and requires computer backup to provide a real-time output of the liquid water, while the latter is labor-intensive in post-flight analysis and also discriminates against the larger droplets because of its small sample volume.

In this paper we describe a hot-wire instrument that is robust, requires at most a simple dry calibration, has a sensitivity of 0.02 g m^{-3}, a response time of the order of 0.05 s and an accuracy of about 5% at 1 g m^{-3}.

2. Design considerations

For the purposes of calibration and analysis, the behavior of a constant-temperature hot wire is more easily interpreted than its constant-current counterpart, so this approach was adopted for the device described herein. The hot-wire element is a hollow cylinder that consists of 2 m of 0.1 mm diameter insulated (varnished) copper wire closely wound as a single layer on a nickel-silver tube of 1.5 mm diameter and 0.16 mm wall thickness. This gives an overall sensitive region 38 mm long and 1.7 mm diameter. A cylinder of this diameter is not only mechanically robust, but is also relatively more sensitive to the heat loss associated with the droplets evaporating than it is to the ventilation heat losses. The power P required to maintain the wire at temperature T_W is

$$P = ldvw[L + c(T_W - T_A)] + \pi lk(T_W - T_A)\,\mathrm{Nu}, \quad (1)$$

where l and d are the length and diameter of the cylinder, v the velocity of the cloud relative to the cylinder, w the liquid water content, L and c the latent and specific heats of water, T_A the ambient air temperature, k the thermal conductivity of air, and Nu the Nusselt number for heat transferred from the wire. The first, or "wet" term, represents the heat required to warm the droplets from T_A to T_W and then evaporate them, and the second, the "dry" term, describes the heat transferred to the cooler air moving past the wire. For thin cylinders moving at

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[1] Manufactured by Johnson-Williams Inc., Mountain View, CA.

[2] Manufactured by Particle Measuring Systems, Boulder, CO.

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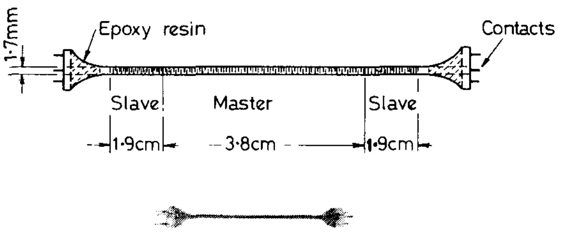

FIG. 1. Details of the hot-wire element. The lead wires to the master are brought out along the rear of the element. A thin coating of epoxy can be applied for extra protection if required.

aircraft speeds, the Nusselt number varies as $d^{0.62}$ whereas the wet term varies as d, so that an increase in d increases the magnitude of the wet term relative to the dry. Since operation of the instrument relies on subtraction of the dry term from the total power to obtain the wet term, greater accuracy can be achieved for larger diameter wires. The diameter cannot be increased indefinitely, however, without considering collection efficiency effects. At aircraft speeds, a 1.7 mm diameter rod collects all droplets >5 μm diameter with better than 95% efficiency (Ranz and Wong, 1952). Although this is sufficient to account for all but a small fraction of the liquid water of most clouds, there are obviously limitations in using a rod 4–5 mm in diameter.

The heat losses that would normally occur through the ends of cylinders of such low aspect ratios are minimized by utilizing similar slave coils (each of resistance equal to half that of the main coil) on either side of the main sensing coil, as shown in Fig. 1. By ensuring that the current in these outer coils exactly duplicates that of the master coil, the axial temperature gradient is substantially reduced (see Fig. 2), and the heat lost in the axial direction is less than 1% of the heat transferred in the radial direction under typical operating conditions. This is, in fact, inherent in the assumption that (1) describes all the heat transferred from the master coil.

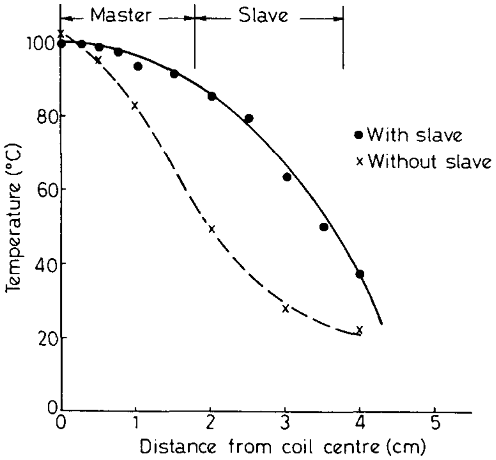

FIG. 2. Temperature distributions along the coil. At the end of the main coil, use of the slave coil raises the temperature from 56 to 90°C.

Another important function of the slave coils is to ensure that the temperature at the outer ends of the sensing coil remains sufficiently high. If the ends are too cool, there is a risk that larger droplets impacting there cannot evaporate before more droplets arrive at the same location, resulting in a saturation effect near the ends. This problem can be alleviated by operating the wire at hotter temperatures, but increasing the temperature to more than about 10°C above the boiling point actually has an adverse effect on evaporation times because of the formation of an insulating vapor layer between droplet and substrate. Hotter temperatures also increase the magnitude of the dry term relative to the wet. The choice of operating temperature therefore represents a compromise between the two conflicting requirements of short evaporation times and low, dry heat losses. This question is discussed in detail in the Appendix, where it is shown that for most cloud situations an operating temperature in the range 80–90°C would be optimal.

For use in supercooled clouds in which graupel or large solid particles could be expected, it is recommended that the surface be coated with a thin layer of epoxy resin. This affords extra protection, and provided it is sufficiently thin to allow the surface temperature to be close to T_W, it does not affect the performance or calibrations.

3. Control circuitry

The circuitry used to maintain the hot wire at constant temperature is fairly standard and is commonly used to control hot-wire anemometers. As shown in the schematic diagram of Fig. 3, the resistance of the hot wire itself is used as the temperature sensor. Any decrease in the wire temperature (and thus resistance) leads to an increase in voltage on the bridge via the amplifier. The consequent heating of the wire increases its resistance and returns the bridge to balance. For the bridge elements shown in Fig. 3, the operating temperature T_W is determined by the set resistors R_1, R_2 and R_3 and lead resistances

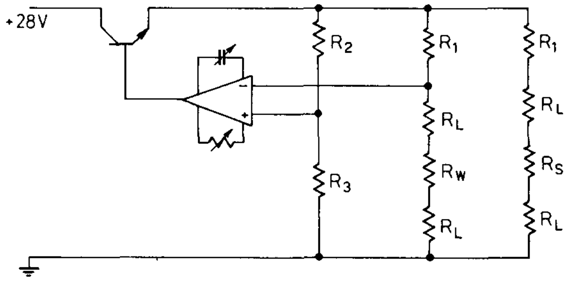

FIG. 3. A schematic diagram of the circuit used to control the hot wire. Full details can be supplied on request.

R_L and is given by

$$T_W = T_0 + (R_3R_1/R_2R_0 - 2R_L/R_0 - 1)/\alpha, \qquad (2)$$

where α is the temperature coefficient of resistivity and R_0 the resistance of the wire at T_0. The power delivered to the hot wire is

$$P = \frac{V^2(R_3R_1/R_2 - 2R_L)}{R_1^2(1+R_3/R_2)^2}, \qquad (3)$$

where V is the voltage across the bridge. The ability to adjust the offset and frequency response of the amplifier is an important feature of this circuit. Both require trimming to the electronic and thermal time constants of the wire if the optimum frequency response of the system is to be achieved. Freymuth (1977) has presented a small-signal analysis for just this type of control loop which enables the offset, gain and frequency response of the amplifier to be selected to optimize the hot-wire frequency response. According to his treatment, the theoretical minimum response time for our wire is of the order of 0.002 s, and a response time of 0.050 s was readily achieved. It should be noted that this response time refers to a step *rise* in the heating load, whether it is the result of an increase in air velocity or liquid water. The response to a step *decrease* in liquid water is much slower however, since it depends on the time required to evaporate the droplets. Further details of this response time will be presented elsewhere (Bradley and King, 1978) but it is sufficient to note here that an overall (i.e., rise or fall) response time of 0.05 s is easily achievable.

4. Experimental tests

a. Dry heat losses

In a 15 cm diameter wind tunnel of 2 m length, the heat lost by the wire was measured as a function of the wire temperature and the air velocity past it. The results of these tests are shown in Fig. 4, where the Nusselt number Nu is plotted against the Reynolds number. The Nusselt number is defined by $\mathrm{Nu} = dJ_H/[k(T_W - T_A)]$ and the Reynolds number by $\mathrm{Re} = \rho_A v d/\mu$, where J_H is the heat flux, and K, ρ_A and μ are, respectively, the thermal conductivity, density and viscosity of air at the mean of the air and hot-wire temperatures. These data can be represented by

$$\mathrm{Nu} = 0.12\ \mathrm{Re}^{0.68}, \qquad (4)$$

showing that the Nusselt numbers are about 12% higher than the recommended curve for flow normal to cylinders (McAdams, 1954). These differences can probably be attributed to the degree of turbulence in the small wind tunnel, since other workers (Comings *et al.*, 1948) have shown that departures of up to 30% from the recommended curve can be effected at high turbulence levels. This hypothesis is also strengthened by the fact that the few data points obtained from aircraft flights lie much closer to the recommended curve. For these reasons, we would not recommended using (4) to account for the dry term but would instead recommend adopting one of the following procedures:

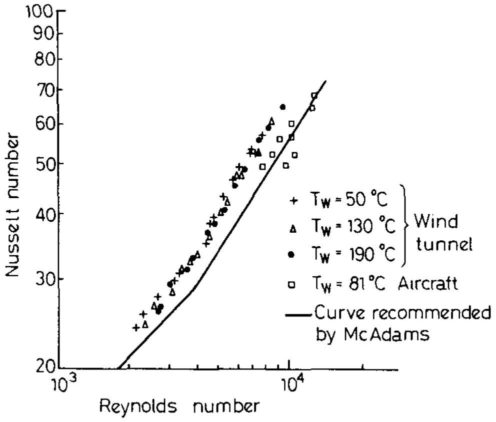

FIG. 4. Heat transfer from the hot wire as a function of Reynolds number. The solid line is an average of those found by other workers. Note that the few aircraft points cluster around this line rather than the wind-tunnel results.

1) Flying in clear air at a given temperature level before entering cloud, noting the power supplied, and storing this value for subtraction from later in-cloud measurements.

2) Carrying out sufficient flight tests to establish a similar curve to the wind-tunnel data of Fig. 4 for the speeds of interest. This latter scheme is regarded as superior.

b. Wet tests

Two types of wet tests were conducted. In the first, a simple, wet wind tunnel capable of producing velocities up to 16 m s^{-1} was constructed by injecting droplets from a stabilized atomizer into an air stream. "Clouds" of liquid water contents up to 6 g m^{-3} could be produced in this fashion. Although these values are artificially high and unlikely to be encountered in nature, the heat loading on the wire produced by this cloud moving past at 16 m s^{-1} is equivalent to that of a 1.5 g m^{-3} cloud sampled at aircraft speeds. The distribution of liquid water in the tube was very nonuniform but remained stable with time. This stability meant that the liquid water sampled by the hot wire could be calculated from the amount of water absorbed by a cylinder of blotting paper of similar dimensions placed in the same location.

Since changes in the diameter of the blotting paper as it absorbed water restricted the accuracy of the experiment to about 15%, further tests were conducted in a cold room using a 2 m^3 supercooled cloud with liquid water contents up to 2.5 g m^{-3}. The hot

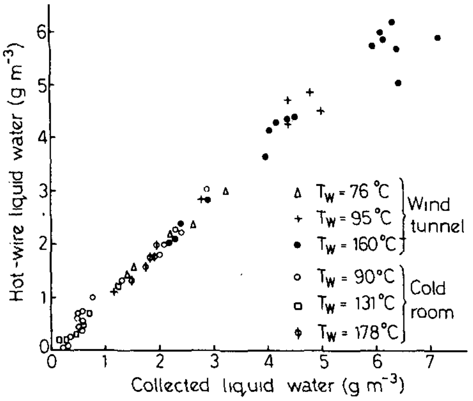

FIG. 5. Results of the wet tests on the hot wire. The ordinate shows values of liquid water content calculated from the wet term of (1), and the abscissa liquid water values obtained from two different collection techniques.

wire was placed at one end of a whirling arm, at the other end of which was a collecting rod of the same dimensions as the hot wire. The experimental procedure was to count and time a given number of revolutions of the arm, noting the power delivered to the wire, and comparing this with the rate of accretion of rime on the collecting rod. The main limitation of this experiment was caused by the stability of the cloud, which tended to dissipate when the arm was set in motion. Despite this, both the liquid water and accretion rates could be determined to within 5% accuracy.

The results of these two sets of experiments are shown in Fig. 5, where it is apparent that the liquid water contents calculated on the basis of the wet term of (1) agree with the measured values within the accuracy of the tests. This implies, of course, that use of this device does not require a wet wind-tunnel calibration, and that its response to liquid water is easily calculated in terms of its geometrical dimensions and operating temperature. While these results have been obtained at comparatively low speeds, we have indirect evidence (King and Handsworth, 1978) that suggests that the device also functions with similar accuracy at aircraft speeds. Further tests, preferably using a wet wind tunnel, should be conducted at typical aircraft cloud penetration speeds to ensure that drops do not blow off the cylinder before they can be evaporated.

5. Accuracy

In using (1) to determine the liquid water, the main errors arise from determining (i) the dry term (equivalent to an error of ±0.01 g m^{-3}), (ii) the power delivered to the wire from a measurement of bridge voltage (1%), (iii) the diameter of the wire (1.5%) and (iv) the true aircraft velocity (1.5%). Other accumulated errors amount to 0.5%. This leads to an overall error of 5% at 2 g m^{-3}, 8% at 0.5 g m^{-3} and 16% at 0.2 g m^{-3}. The sensitivity of the device is of the order of 0.02 g m^{-3}, and is limited by observable changes in the dry term. All of these values were calculated for a wire operated near 100°C at speeds of 60 m s^{-1}. If the device were used at speeds much less than this, the errors would rise approximately as $v^{-\frac{1}{2}}$.

6. Conclusions

A hot-wire cylinder of dimensions 1.7 mm×4 cm with contiguous slave coils has been found to be useful for measuring cloud liquid water content from aircraft. Its chief advantage is that it requires no wet calibration. The device has a sensitivity of about 0.02 g m^{-3}, a response time of better than 0.050 s and an accuracy of about 5% at 1 g m^{-3}. It is robust, easily replaceable, and has been flown for a total of 150 h with minimal developmental problems.

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APPENDIX

Operating Temperature of the Wire

It is important that the wire be operated at a temperature high enough to prevent the accumulation of water. One method of investigating this aspect is to examine the fraction of the surface area of the wire covered with droplets at any time; values approaching unity would be an indication that saturation was occurring. We first look at the time taken for a droplet to evaporate.

Consider a droplet that impacts and remains at the stagnation point of the cylinder and does not wet the surface but beads up as a hemisphere. (By ignoring the effects of spread and ventilation we will obtain upper limits on the evaporation times and conservative estimates of the fraction of the wire surface covered with liquid.) Since the thermal conductivity of the wire and water is much greater than that of air, we can assume with little error that the droplet remains close to the wire temperature during the evaporation process. Under this assumption, the mass flux J_m is then

$$J_m = 2\pi a(t) D \Delta\rho, \qquad \text{(A1)}$$

where a is the droplet radius at time t, D the diffusivity of water vapor in air, and $\Delta\rho$ the difference in saturated vapor densities over water at the wire and environmental temperatures. It then follows that the droplet radius at time t is

$$a(t) = a_0(1 - t/\tau)^{\frac{1}{2}}, \qquad \text{(A2)}$$

where a_0 is its initial radius and the evaporation time τ is given by

$$\tau=\frac{a_0^2\rho_w}{2D\Delta\rho},\tag{A3}$$

where ρ_w is the density of water.

We now consider a long-time t', in which the rod collects $nldvt'$ droplets, where n is the total droplet concentration, and the size spectrum is described by $g(a)$ where

$$\int_0^\infty \frac{dg}{da}da=1.$$

Of the total number of droplets which hit in t', those in the size range da will be on the cylinder for a time $\tau(a)$, occupying an average area during this time of $\pi a^2/2$. The fraction of the total front surface of the wire that this group occupies on average is thus $\tau(a)\pi a^2/2t'ld$, so that the total fraction occupied on average by the whole range of sizes of the total $nldvt'$ is

$$f=nldvt'\int_0^\infty \frac{dg}{da}\frac{\tau(a)\pi a^2}{2t'ld}da.\tag{A5}$$

Using (A3), we simplify this to

$$f=\frac{nv\pi\rho_W}{4D\Delta\rho}\int_0^\infty \frac{dg}{da}a^4da,\tag{A6}$$

$$=\frac{nv\pi\rho_W}{4D\Delta\rho}\overline{(a^4)},\tag{A7}$$

where the bar signifies an average over the droplet population. Now $\overline{(a^4)}$ for most cloud droplet distributions other than strongly skewed ones is less than $2(\bar{a})^4$, so we have

$$f<nv\pi\rho_W(\bar{a})^4/2D\Delta\rho.\tag{A8}$$

It is useful to consider this in terms of liquid water contents, i.e.,

$$w=\tfrac{4}{3}\pi\rho_W n\overline{(a^3)}.\tag{A9}$$

For distributions which are symmetric about the mean and have a dispersion of 0.3, we then have

$$\overline{(a^3)}=1.27(\bar{a})^3,\tag{A10}$$

and using (A9) and (A10) we find

$$f<\frac{0.17vw^{\frac{4}{3}}}{D\Delta\rho\rho_W{}^{\frac{1}{3}}n^{\frac{1}{3}}}.\tag{A11}$$

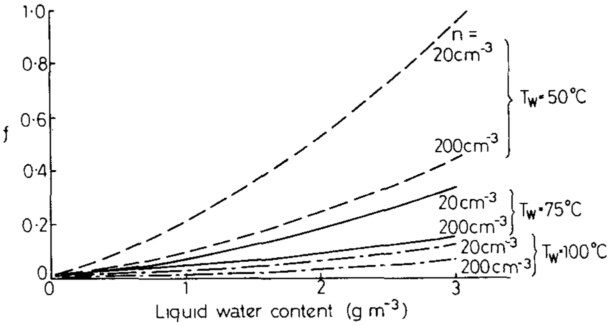

FIG. A1. The fraction of the front surface of the wire covered in droplets as a function of liquid water, operating temperature and droplet concentrations.

This function is plotted in Fig. A1 for $v=60$ m s^{-1} as a function of liquid water content for two values of n, and for three values of wire operating temperature. It is apparent that for a given liquid water content a maritime droplet spectrum is more likely to saturate than a continental one, and that a temperature in excess of 75°C is required to ensure that saturation is not approached in maritime clouds of up to 3 g m^{-3}. This requirement, in fact, overrides the conflicting need for lower temperatures to reduce the magnitude of the dry term. It is not recommended that the wire be operated in excess of 100°C because the boiling point of water at some altitudes of interest to cloud physicists can be as low as 90°C. At these levels, evaporation at temperatures above 100°C is likely to be slower than at 100°C because of the formation of an insulating vapor layer. An operating temperature somewhere between 80 and 90°C is probably optimal.

REFERENCES

Bradley, S., and W. D. King, 1978: Frequency response of the CSIRO liquid water probe. (To be submitted to *J. Appl. Meteor.*)

Comings, E. W., J. J. Clapp and J. F. Taylor, 1948: Air turbulence and transfer processes. Flow normal to cylinders. *Ind. Eng. Chem.*, **40**, 1076–1082.

Freymuth, P., 1977: Frequency response and electronic testing for constant-temperature hot-wire anemometers. *J. Phys.*, **E10**, 705–710.

King, W. D., and R. J. Handsworth, 1978: Total droplet concentrations and average droplet sizes from simultaneous liquid water content and extinction measurements. Submitted to *J. Appl. Meteor.*

McAdams, W. H., 1954: *Heat Transmission.* McGraw-Hill, 532 pp.

Merceret, F. J., and T. L. Schricker, 1975: A new hot-wire liquid cloud water meter. *J. Appl. Meteor.*, **14**, 319–326.

Ranz, W. E., and J. B. Wong, 1952: Impaction of dust and smoke particles on surface and body collectors. *Ind. Eng. Chem.*, **44**, 1371–1379.